ties like 0.1 microfarad fail to influence the vibrator though to the ear the sound is quite loud. The capacity should have to be increased to 10 or 20 microfarads for equal effects on telephone and vibrator. In another experiment the closed circuit gave 20 scale-parts. The insertion of 4 microfarads decreased this to 4 scale-parts, which is again a demand of about 20 microfarads for an equality of behavior. On the other hand, while the telephone responds for a phenomenally small capacity, it soon ceases to increase in loudness (for 1, 2, 3, 4, mf., or resistances), whereas the deflections of the vibrator increase regularly.

[1] Advance note from a Report to the Carnegie Institution of Washington, D. C.

[2] These PROCEEDINGS, **5**, 211–217, (1919).

[3] These PROCEEDINGS, **4**, 328–333, (1918). Carneg. Publ. No. 249, **3**, 1919, chap. v.

ON THE PRESSURE VARIATION OF SPECIFIC HEAT OF LIQUIDS

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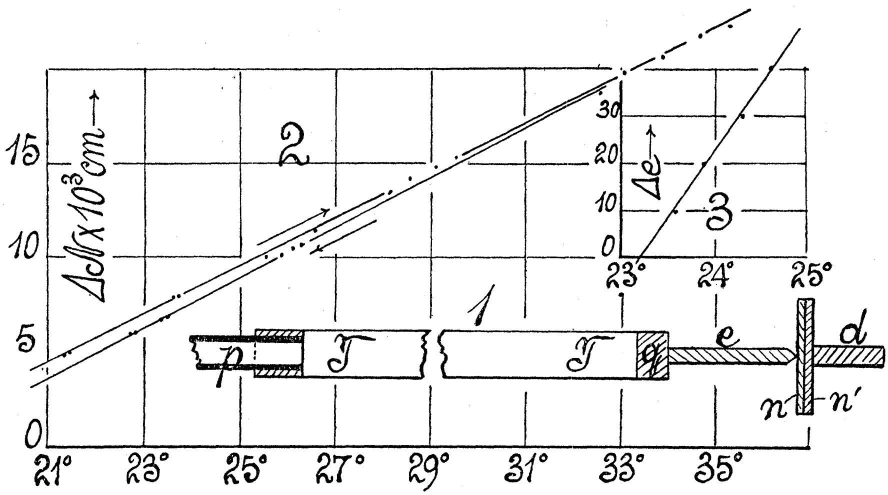
1. *Introductory.*—The measurement of the specific heat of a liquid in its relation to pressure is surrounded by so many difficulties, that any method which gives a fair promise of success deserves to be carefully scrutinized. During the course of my recent work on interferometry, I have had this in view, and the plan which the present paper proposes is particularly interesting as it seems to be quite selfcontained.

2. *Equations.*—If θ, p, ρ, c, denote the absolute temperature, the pressure, the density, and the specific heat at constant pressure, respectively, of the liquid, and if $\alpha' = (dv/v)/d\theta$ is its coefficient of volume expansion, the relation of these quantities may be expressed by the well known thermodynamic equation

$$d\theta/dp = \alpha'\theta/J\rho c \tag{1}$$

where J is the mechanical equivalent of heat, and the transformation is along an adiabatic. The main difficulty involved would therefore be the measurement of the temperature increment; for dp could be read off on a Bourdon gauge after a partial stroke of the lever of my screw compressor, with facility. It is my purpose to find $d\theta$ by the displacement interferometer. To fix the ideas; suppose the liquid in

question is introduced into a long steel tube TT, figure 1, and that the tubulure p conveys the increments of pressure dp. The end p is rigidly fixed. The other end q of the tube is free to move. By aid of the stylus, e, the elongation is registered on the plate n of a contact lever read by interferometry, the lever being identical in construction with the apparatus described in my paper on magnetic elongation.[1] Thus the interferometer will indicate the elongations due both to the pressure increment and to the corresponding temperature increment of the suddenly compressed liquid, and it becomes a question to what degree the two may be adequately separated. If Δl, Δp, $\Delta\theta$ are corresponding increments of the length, l, of the tube and the



pressure and temperature of its liquid content, we may write successively, if $\Delta l = \Delta l' + \Delta l''$,

$$\Delta l'/l = (r_1^2/3k(r_2^2 - r_1^2))\ \Delta p = \beta\Delta p, \text{ say,} \tag{2}$$

$$\Delta l''/l = \alpha\Delta\theta \tag{3}$$

where α is the coefficient of expansion, k the bulk modulus, r_1 and r_2 the internal and external radius of the steel tube of length l. Hence

$$\alpha\Delta\theta = \Delta l/l - \beta\Delta p \tag{4}$$

and equation (1) becomes

$$d\theta/dp = \frac{\Delta l/(l\Delta p) - \beta}{\alpha} = \frac{\alpha'\theta}{J\rho c} \tag{5}$$

or

$$c = (\alpha'\theta/J\rho)\ (\alpha/(\Delta l/(l\Delta p) - \beta) \tag{6}$$

Hence c may be found from observations of Δl and Δp, provided α' and ρ, α and β are sufficiently known.

3. Measurement of the Pressure Coefficient β.—For this purpose the tube TT, figure 1, is placed in a water jacket of constant temperature, and β found by internal pressure, directly. Experiments of this kind were contributed with some detail in an earlier paper.[2] The method then used consisted in finding β from the displacement of the spectrum ellipses under known conditions; but the present method of the contact lever and achromatic fringes may be considered preferable, particularly if the tube contains water, for which the thermal discrepancy is small.

Moreover, since $\Delta\theta$ is primarily aimed at, β should be made as small as possible. This may be done by selecting relatively thick walled tubes of small external diameter. A few data are here desirable. Using an ocular micrometer plate 1 cm. long with scale parts of 0.01 cm. each and fringes of moderate size (one or two scale parts in width) we may write as in the preceding paper (l. c.)

$$\Delta l/l = 3 \times 10^{-7}\,\Delta e \tag{7}$$

where Δe is the displacement of the achromatic fringe on the ocular scale corresponding to the elongation $\Delta l/l$.

Hence for steel tubes ($k = 1.8 \times 10^{12}$) the following data apply

TUBES	$2r_1$	$2r_2$	$\beta \times 10^7$	$\Delta e'/\Delta p$
	cm.	*cm.*		
I	1.0	0.9	8.0	2.7
II	1.0	0.8	3.3	1.1
III	0.7	0.6	5.2	1.7
IV	0.7	0.5	1.9	0.63

β being reckoned per atmosphere.

4. Measurement of α.—For this purpose the tube TT is to be clean and empty, the nozzle p removed and a long-stemmed thermometer passed from end to end of the tube, through the end p. Externally the tube is surrounded by a coil of wire for electric heating and appropriately jacketed. Measurements made in this way with a brass tube are given in figures 2 and 3 and they would have been quite satisfactory if the tube had been properly protected from loss by radiation. (ΔN is read off on the displacement micrometer at an interferometer mirror; Δe in the ocular scale).

If for the steel tube, $\alpha = 10^{-5} \times 12$, equations (3) and (6) then give us

$$\Delta\theta = 0.025\,\Delta e$$

or about 40 scale parts of the ocular micrometer per degree of temperature.

5. Liquids.—It remains to select suitable liquids for experiment. For water at 27°, $d\theta/dp = 0.0019$; for ethylic alcohol at about 18°, $d\theta/dp = 0.017$; for ether at 18°, $d\theta/dp = 0.028$, pressures being measured in atmospheres. Thus for the four tubes specified in §3, the respective displacements in the ocular micrometer would be (per atmosphere)

I	$\Delta e' = 2.7$	water:	$\Delta e'' = 0.08$
II	1.1	alcohol	$\Delta e'' = 0.68$
III	1.7	ether	$\Delta e'' = 1.14$
IV	0.63		

The case of water may be dismissed for here the thermal displacement per atmosphere, $\Delta e''$, is a small fraction of the elastic displacement in the ocular micrometer. But alcohol and ether show satisfactory conditions. Thus a sudden half turn of the lever of the screw compressor producing 100 atmospheres would displace the fringes, in case of tube III and ether, 173 scale parts elastically and 114 scale parts thermally, together 287 scale parts. Stops of 30 atmospheres would be advisable. Tube IV with 63 and 114 scale parts respectively is even more advantageous.

It remains to estimate the diminution of $\Delta\theta$ owing to the partition of heat between the liquid and the tube. If $\Delta\theta'$ is the increment of the combined system of liquid and tube the ratio will be

$$x = \frac{\Delta\theta'}{\Delta\theta} = 1/\left(1 + \frac{r_2^2 - r_1^2}{r_1^2}\frac{c'\rho'}{c\rho}\right)$$

if c' and ρ' are the specific heat and density of the solid. These ratios x for the tubes and liquids in §3 and the corrected $\Delta e''$ are easily tabulated. Tubes of the type I are unsatisfactory. In case of tubes of the types II or III the thermal displacement would be but about 5% of the elastic displacement in case of water; but in case of alcohol 25 to 35%, and of ether 35 to 55%. In tubes of the type IV the advantages of thicker walls and small external diameter have again decreased. The problem of selecting the best tube admits of general solution.

If we combine equations (4) and (8) and put

$$A = \Delta l/l;\quad B = \Delta p/3k;\quad C = \rho'c'/\rho c;\quad y = r_2^2 - r_1^2/r_1^2$$

the result is

$$\alpha\Delta\theta' = \frac{A - B/y}{1 + Cy} \tag{9}$$

Here y is the ratio of solid and liquid sections and we inquire what value of y will make $\Delta\theta'$ a maximum provided A, B, C are constant. If the thermal and elastic elongations are to be equal $A = 2B$. Differentiating (9) and reducing:

$$1/y = C(\sqrt{1 + A/BC} - 1) \tag{10}$$

and since y must be positive the radical is positive. Now if $A = 2B$, for example, the ratio of diameters $2r_1$ to $2r_2$ would in all cases have to exceed 0.65. If $A = 3B$, the case of water remains nearly the same, but for ether and alcohol the diameter ratio approaches 0.9.

[1] These PROCEEDINGS, 5, 1919, (267–272).
[2] Carnegie Publ. No. 249, 1917, pp. 84–94.

STUDIES OF MAGNITUDES IN STAR CLUSTERS, IX. THE DISTANCES AND DISTRIBUTION OF SEVENTY OPEN CLUSTERS

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The question of whether globular clusters are really or only apparently absent from the mid-galactic segment makes the study of the distances of open clusters particularly important. These objects are relatively near to the galactic circle, and many appear to be at such great distances along the plane as to support the hypothesis that obstructing matter is insufficient to occlude globular clusters in mid-galactic regions.[1] On the other hand there is evidence that globular clusters actually may not be absent from low galactic latitude,[2] and the following discussion of open clusters and other relevant factors suggests that the question must be considered an unsettled one for the time being.

Although the question of the reality of the region of avoidance affects but little the general conclusions reached in earlier papers regarding globular clusters, spiral nebulae, and the Galaxy, two modifications should be made to previously suggested interpretations,[3] in case we demonstrate the existence of much obstructing matter along the galactic